РОЗДІЛ 9
МАТЕМАТИЧНІ МЕТОДИ, МОДЕЛІ ТА ІНФОРМАЦІЙНІ ТЕХНОЛОГІЇ В ЕКОНОМІЦІ

UDC 519.865: 338.518

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COURNOT AND STACKELBERG EQUILIBRIA IN THE AKERLOF MODEL

This paper investigates Cournot and Stackelberg equilibria in the Akerlof model. For this purpose, there is constructed a model of duopoly producers of high-quality and low-quality products, which compete in conditions of information asymmetry. The demand curve is described by a function with constant elasticity. In the model identified the corresponding states of equilibrium and conducted their comparative analysis. It is found that optimal for duopolists is the Stackelberg equilibrium, when leader – a manufacturer of high-quality product. It is determined that adverse selection is valid in all states of equilibrium.
Key words: quality asymmetry, Akerlof asymmetry, isoelastic demand function, adverse selection.

Problem setting and its connection with important scientific and practical tasks. The basis of classical economic theory is the assumption of the completeness and accuracy of information held by economic agents. Based on this assumption concludes theoretical possibility of rational behavior of economic agents and achieving economic efficiency by Pareto. However, this assumption does not correspond to economic reality and a lot of research in the XX century were devoted to the impact of information and other forms of asymmetry in market processes. The result of this research was the first in the XXI century, the Nobel Prize in economics that was awarded Akerlof, Spence and Stiglitz for the development of the theory of markets with asymmetric information. Akerlof studied the information asymmetry between buyer and seller about quality of products, Stiglitz – information asymmetry in insurance market, Spence – information asymmetry between employer and employee regarding the employee's qualifications.

Recent research and publications analysis. Information asymmetry can substantially affect on the market equilibrium. Impact of asymmetry effects in oligopoly models studied in many works.

The paper [1] studies Akerlof's market for lemons in a new way. The author construct mixed Perfect Bayesian Nash equilibria in which all qualities are sold on the market even if the seller's strategy set is reduced to prices. The paper [2] examines markets where the characteristics or decisions of certain agents are relevant but not known to their trading partners. Assuming exclusive transactions, the environment is described as a continuum economy with indivisible commodities. In [3] have compared Bertrand and Cournot equilibria in a differentiated duopoly with linear demand and cost functions. Focusing on the case of substitute goods, author show that both the efficient firm's profits and industry profits are higher under Bertrand competition when asymmetry is strong and/or products are weakly differentiated. The paper [4] examines the asymptotic inefficiency of Stackelberg markets with incomplete information. In the model firms make their quantity choices based on limited information and their output choices are likely to deviate from those optimal under complete information. It found that Stackelberg markets with incomplete information are asymptotically inefficient with probability one. In [5] have investigated the connection between cost asymmetries and the sustainability of collusion within the context of a infinitely repeated Cournot duopoly. Shown that regardless of the degree of cost asymmetry, at least some collusion is always sustainable if firms are patient enough. In [6], on base of econometric model, investigated impact of the cost asymmetry on the behavior of firms in duopoly market.

The paper [7] examines how incentives for two duopolists to honestly share information change depending upon the nature of competition (Cournot or Bertrand) and the nature of the information structure. In [8] have investigated the problem of information sharing in duopoly games with heterogeneous costs. The paper attempts to analyze the question whether and to what extent in a differentiated product market, firms with different cost functions have incentives for sharing private information about demand or cost.

Formulation of research objectives. The literature examined usually the influence of one type of asymmetry: costs, prices, quality, information etc. It is of interest to investigate the equilibrium in the duopoly model of manufacturers in conditions of impact of quality asymmetry, Akerlof asymmetry (availability of information about the quality) and the Stackelberg asymmetry (availability of information about a competitor). Thus, the **goal of this article** is to analyze Cournot and Stackelberg equilibria in the Akerlof model.

The basic results and their justification. Consider two local markets, indexed as **0** (low quality) and **1** (high quality). Local markets are functioning in the conditions of information symmetry – consumers have full information about the quality of goods.

There are an asymmetric costs between markets, due to the asymmetry of the quality of goods. The low-quality manufacturer has constant marginal costs, denoted z. The high-quality manufacturer has constant marginal costs $z \cdot k$, where $k > 1$ – coefficient of quality asymmetry, reflecting different quality of the goods.

Demand on local markets described by isoelastic demand functions. Assume the demand function of low-quality product: $p_0 = 1/q_0$, high-quality product: $p_1 = k/q_1$, where p_0, p_1 denotes market prices and q_0, q_1 denote the outputs of the duopolists. By investing in quality, the high-quality manufacturer increases the value of goods to consumers, and they are willing to pay for the same amount k times greater. Therefore, high-quality manufacturer expects a corresponding increase unit profit.

Let us assume that low-quality manufacturer entered to the quality market and advertise own product as a quality. The result is a new market duopoly where consumers can no longer distinguish quality of products due to the Akerlof asymmetry. Demand function to a duopoly market will be in the form: $p = (1-\alpha)\cdot p_0 + \alpha \cdot p_1 = (k+1)/(q_0+q_1)$, where $\alpha = q_1/(q_0+q_1)$ – proportion of high-quality goods, $(1-\alpha)$ – proportion of low-quality goods.

Since the market price is in the range $p_0 < p < p_1$, then in conditions of Akerlof asymmetry the low-quality manufacturer wins and a high-quality manufacturer loses. This results in adverse selection, ousting of high-quality goods from the market and market disappearance [9].

Profit functions of duopolists

$$F_0 = \frac{q_0 \cdot (k+1)}{q_0+q_1} - q_0 \cdot z \to \max_{q_0},$$

$$F_1 = \frac{q_1 \cdot (k+1)}{q_0+q_1} - q_1 \cdot z \cdot k \to \max_{q_1}.$$

Putting the first derivatives $dF_0/dq_0 = 0$ and and solving for q_0, q_1 one obtains:

$$q_0^* = \sqrt{\frac{q_1 \cdot (k+1)}{z}} - q_1,$$

$$q_1^* = \sqrt{\frac{q_0 \cdot (k+1)}{z \cdot k}} - q_0,$$

which are the reaction functions. Second derivatives $d^2F_0/dq_0^2 < 0$, $d^2F_1/dq_1^2 < 0$, it means that a profit functions achieves its maximum.

Using the standard procedure, we will define Cournot and Stackelberg equilibria in this model. Let us analyze three equilibria: Cournot (C); Stackelberg, where the leader – low-quality manufacturer (S_0); Stackelberg, where the leader – high-quality manufacturer (S_1). For each equilibrium we will find the Akerlof point – coefficient of quality asymmetry in which the market disappears – splits into two local markets. A sign of Akerlof point we assume zero output of goods or zero profit or loss of stability of an equilibrium state. The results are shown in Table 1.

Let us do a comparative analysis of equilibria.

To determine Akerlof point in the Cournot equilibrium, consider a two-dimensional map

$$q_0^*(t+1) = \sqrt{\frac{q_1(t)\cdot(k+1)}{z}} - q_1(t),$$

$$q_1^*(t+1) = \sqrt{\frac{q_0(t)\cdot(k+1)}{z\cdot k}} - q_0(t), \quad (1)$$

It is known from [10], when $k^A = 3 + 2\cdot\sqrt{2}$ the fixed point (q_0^C, q_1^C) of two-dimensional map (1) loses stability.

At the Stackelberg equilibrium $(q_0^{S_0}, q_1^{S_0})$ the low-quality goods completely supplant the high-quality goods at the Akerlof point: $k^A = 2$. In this case, the bargaining power of low-quality manufacturer so great (in his favor all asymmetry), that high-quality manufacturer will hold on only for $k < 2$.

To determine Akerlof point at the Stackelberg equilibrium $(q_0^{S_1}, q_1^{S_1})$ we must set the minimum level of the share of high-quality goods, where the high-quality manufacturer leaves the market.

Let us a compare the equilibrium outputs:

$$q_0^{S_0} > q_0^C > q_0^{S_1} > q_1^C > \{q_1^{S_0}, q_1^{S_1}\}.$$

The relationship between outputs of high-quality manufacturer at Stackelberg equilibriums depend on the coefficient of quality asymmetry:

Table 1

Cournot and Stackelberg equilibria in the Akerlof model

Equilibrium	Cournot	Stackelberg S0	Stackelberg S1
k	$1<k<3+2\cdot\sqrt{2}$	$1<k<2$	$k>1$
q_0^e	$\frac{k}{z\cdot(k+1)}$	$\frac{k\cdot(k+1)}{4\cdot z}$	$\frac{(k+1)\cdot(2\cdot k-1)}{4\cdot z\cdot k^2}$
q_1^e	$\frac{1}{z\cdot(k+1)}$	$\frac{(k+1)\cdot(2-k)}{4\cdot z}$	$\frac{k+1}{4\cdot z\cdot k^2}$
$Q^e = q_0^e + q_1^e$	$\frac{1}{z}$	$\frac{k+1}{2\cdot z}$	$\frac{k+1}{2\cdot z\cdot k}$
F_0^e	$\frac{k^2}{k+1}$	$\frac{k\cdot(k+1)}{4}$	$\frac{(k+1)\cdot(2\cdot k-1)^2}{4\cdot k^2}$
F_1^e	$\frac{1}{k+1}$	$\frac{(k+1)\cdot(2-k)^2}{4}$	$\frac{k+1}{4\cdot k}$
$r_0^e = F_0^e/(q_0^e\cdot z)$	k	1	$2\cdot k-1$
$r_1^e = F_1^e/(q_1^e\cdot z\cdot k)$	$\frac{1}{k}$	$\frac{2-k}{k}$	1
$p_0(q_0^e)$	$\frac{z\cdot(k+1)}{k}$	$\frac{4\cdot z}{k\cdot(k+1)}$	$\frac{4\cdot z\cdot k^2}{(k+1)\cdot(2\cdot k-1)}$
p^e	$z\cdot(k+1)$	$2\cdot z$	$2\cdot z\cdot k$
$p_1(q_1^e)$	$z\cdot k\cdot(k+1)$	$\frac{4\cdot z\cdot k}{(k+1)\cdot(2-k)}$	$\frac{4\cdot z\cdot k^3}{k+1}$
α^e	$\frac{1}{k+1}$	$\frac{2-k}{2}$	$\frac{1}{2\cdot k}$
k^A	$3+2\cdot\sqrt{2}$	2	–
$\alpha^e(k^A)$	14,6%	0%	–

$sign\left(q_1^{S_0} - q_1^{S_1}\right) = sign(k - \Phi)$, where $\Phi = \left(1+\sqrt{5}\right)/2$ – „golden" ratio.

A comparison of equilibrium profits:

$$F_0^{S_1} > F_0^{S_0} > F_0^C > F_1^{S_1} > F_1^C > F_1^{S_0}. \qquad (2)$$

Interestingly, that the equilibrium profits (2) invariant with respect to costs and depend only on the coefficient of quality asymmetry. From (2) we see that the availability of information about the production strategy of competitor does not help the high-quality manufacturer to overcome information asymmetry of Akerlof and make more profit. Thus, in this model, the Akerlof asymmetry has a stronger effect compared with Stackelberg asymmetry.

Dynamics of equilibrium profits depending on coefficient of quality asymmetry presented in Figure 1.

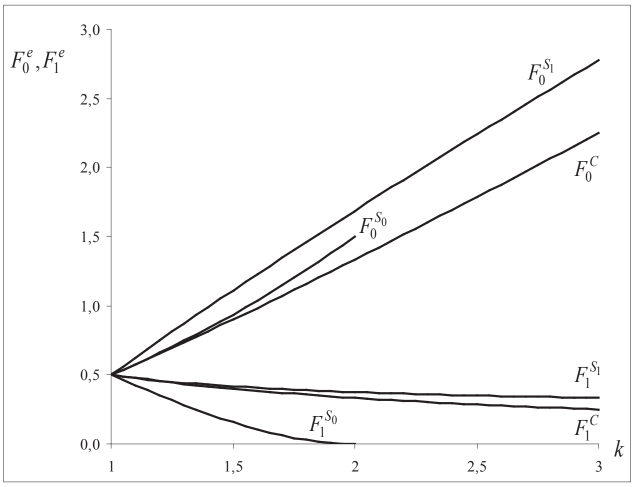

Fig. 1. Dynamics of equilibrium profits

Increased investment in quality, paradoxically, lead to the ousting of high-quality goods from the market and increase profit of low-quality manufacturer. If consumers can not distinguish the quality of goods, the high-quality manufacturer will have to either leave the market or reduce quality. As a result, it becomes a duopoly market of low-quality goods. Thus, this model illustrates the adverse selection with information asymmetry.

Also, from (2) follows that the equilibrium $\left(F_0^{S_1}, F_1^{S_1}\right)$ is optimal for both duopolists. As a rule, the leader in Stackelberg model gains higher than the follower [11]. And this is normal, as the leader has more information about a competitor. However, in the Akerlof model the profit of low-quality manufacturer in the follower position will more than a on leader position.

Equilibrium returns on transport costs:

$$r_0^{S_1} > r_0^C > r_0^{S_0} = r_1^{S_1} > r_1^C > r_1^{S_0}.$$

Dynamics of equilibrium returns on transport costs depending on coefficient of quality asymmetry presented in Figure 2.

From Table 2 we see that in conditions of impact of Akerlof asymmetry the high-quality manufacturer can achieve 100% return on transport costs on the position of the leader Stackelberg. Equilibrium return on transport costs of leaders is the same and equal 100%.

Let us to compare equilibrium market prices and industry outputs:

$$p^{S_0} < p^C < p^{S_1},\ Q^{S_0} > Q^C > Q^{S_1}. \qquad (3)$$

From (3) we see that the leader (low-quality manufacturer) saturates the market with cheap and low

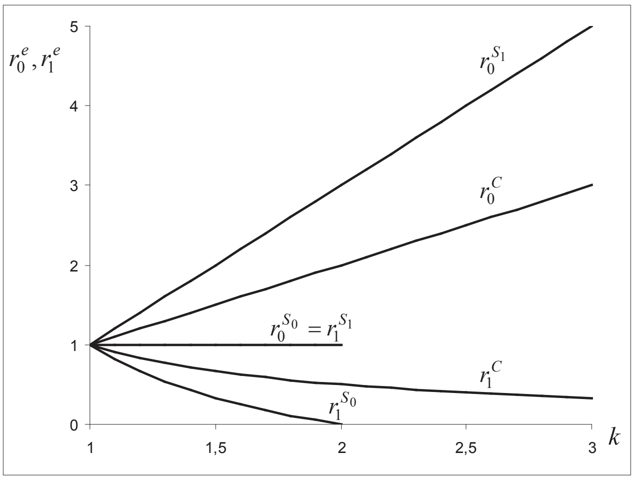

Fig. 2. Dynamics of equilibrium returns on transport costs

quality goods. Leadership of the high-quality manufacturer leads to a reduction in the volume of supply of low-quality goods and an increase in market price.

Note that for all equilibria are accomplished conditions: $p_0\left(q_0^e\right) < p^e < p_1\left(q_1^e\right)$.

All equilibrium states for $k = \Phi$ are presented in Figure 3.

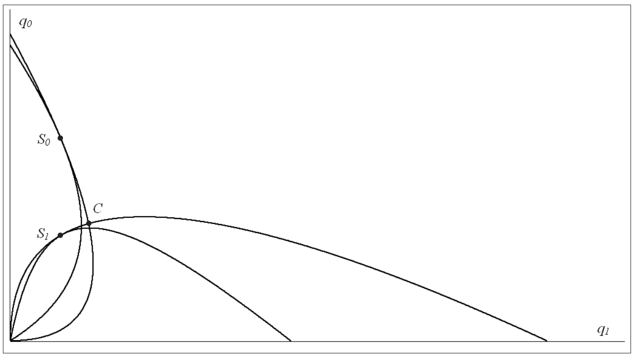

Fig. 3. Cournot and Stackelberg equilibria in the Akerlof model

We illustrate the obtained results on the numerical example (Table 2). Data: $k = \Phi$, $z = 0{,}5$. The calculations confirm the obtained analytical results.

Dynamic of Cournot equilibrium outputs of manufacturers depending on coefficient of quality asymmetry for $k \in (1;\ 6{,}25]$ presented in Figure 4. From

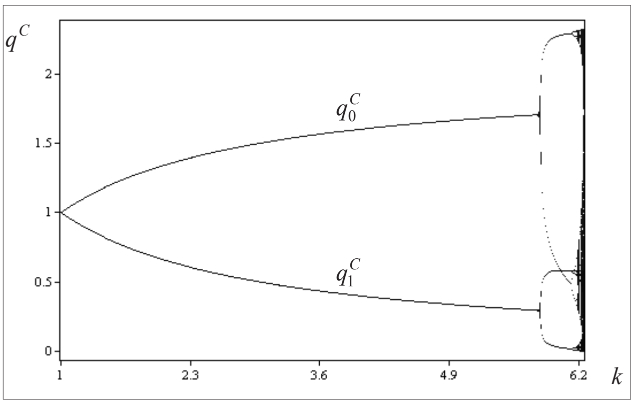

Fig. 4. Equilibrium trajectories of manufacturers at the Cournot equilibrium

Table 2

Numerical example

Equilibrium	Cournot	Stackelberg S_0	Stackelberg S_1
q_0^e	1,236	2,118	1,118
q_1^e	0,764	0,500	0,500
Q^e	2,000	2,618	1,618
F_0^e	1,000	1,059	1,250
F_1^e	0,382	0,095	0,405
r_0^e	1,618	1,000	2,236
r_1^e	0,618	0,236	1,000
$p_0(q_0^e)$	0,809	0,472	0,894
p^e	1,309	1,000	1,618
$p_1(q_1^e)$	2,118	3,236	3,236

Figure 4 we see that in Akerlof point $k^{\Lambda} = 5,83$, the Cournot equilibrium loses stability and arise bifurcation.

Conclusions and prospects for further research. The analysis found that in the Akerlof model the low-quality manufacturer more produces, sells at a higher price and get more profit. The high-quality manufacturer invest in quality, but the return on investment takes the low-quality manufacturer. In conditions of information asymmetry the profit of low-quality manufacturer at Stackelberg equilibrium in the position of follower higher than in the leader position. It was found that the optimal for both duopolists is Stackelberg equilibrium, when leader – high-quality manufacturer.

In the future supposed simulation of equilibrium in the Akerlof model under impact of other asymmetries.

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РІВНОВАГИ КУРНО І ШТАКЕЛЬБЕРГА В МОДЕЛІ АКЕРЛОФА

Резюме
У статті досліджуються стани рівноваги Курно та Штакельберга в моделі Акерлофа. Для цього побудовано модель дуополії виробників якісного та неякісного товарів, які конкурують в умовах інформаційної асиметрії. Крива попиту описується функцією з постійною еластичністю. У моделі визначено відповідні стани рівноваги та проведено їх порівняльний аналіз. Отримано, що оптимальним для дуополістів є рівновага Штакельберга, коли лідер – виробник якісного товару. Визначено, що негативний відбір діє в усіх станах рівноваги.
Ключові слова: асиметрія якості, асиметрія Акерлофа, функця попиту з постійною еластичністю, негативний відбір.

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РАВНОВЕСИЯ КУРНО И ШТАКЕЛЬБЕРГА В МОДЕЛИ АКЕРЛОФА

Резюме
В статье исследуются состояния равновесия Курно и Штакельберга в модели Акерлофа. С этой целью построена модель дуополии производителей качественного и некачественного товаров, конкурирующих в условиях информационной асимметрии. Кривая спроса описывается функцией с постоянной эластичностью. В модели определены соответствующие состояния равновесия и проведен их сравнительный анализ. Получено, что оптимальным для дуополистов является равновесие Штакельберга, когда лидер – производитель качественного товара. Определено, что негативный отбор действует во всех состояниях равновесия.
Ключевые слова: асимметрия качества, асимметрия Акерлофа, функция спроса с постоянной эластичностью, негативный отбор.